Journal of Life Sciences Research
Vol. 7, No. 1, 8-12, 2020
ISSN(E) 2408-9184 / ISSN(P) 2518-0126
DOI: 10.20448/journal.504.2020.71.8.12

Mathematical Model Governing the Predator-Prey Relationship in Lake Turkana (Kenya)

G.W. Gachigua[1]
P.M. Wanjau[2]

[1,2]*School of Mathematics and Actuarial Science, The Technical University of Kenya, Nairobi, Kenya.*
[1]*Email: wambuigachigua@gmail.com*

(Corresponding Author)

Abstract

Predator-Prey relationship is a very fundamental aspect of every ecosystem. The associations that exist between different organisms in the ecosystem influence the survival of organisms and the functioning of the ecosystem as a whole. Lake Turkana is an understudied, permanent desert, fresh water lake located in north western, Kenya. Some of the species found in the lake include the Kingfishers and the Palegic fish. In this study the predator-prey relationship between the Kingfishers and the Pelagic fish (Pyrenean minnow) in the Lake Turkana has been analyzed qualitatively. A model of first order differential equations is proposed and studied after making certain simplifying assumptions. The analytical result showed that the system has two equilibriums, namely the trivial equilibrium and the endemic equilibrium which are stable. Numerical simulation was performed and analysis done to show the relationship between the two species. Results obtained from the simulation compares well with data obtained from Turkana Fisheries Company. The changes in the predator-prey species in the lake shows that predation has the balance of nature that exists in this eco-system.

Keywords: Kingfisher, Pelagic fish, Prey, Predator, Equilibrium points, Stability.

Citation | G.W. Gachigua; P.M. Wanjau (2020). Mathematical Model Governing the Predator-Prey Relationship in Lake Turkana (Kenya). Journal of Life Sciences Research, 7(1): 8-12.
History:
Received: 24 March 2020
Revised: 27 April 2020
Accepted: 21 May 2020
Published: 6 June 2020

Publisher: Asian Online Journal Publishing Group

Acknowledgement: Both authors contributed to the conception and design of the study.
Funding: This study received no specific financial support.
Competing Interests: The authors declare that they have no conflict of interests.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study was reported; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained.
Ethical: This study follows all ethical practices during writing.

Contents

Contribution of this paper to the literature
This study is applicable by the Kenya Wildlife Service to predict the population of the species in the lake in future policy making.

1. Introduction

Lake Turkana is an understudied rift valley lake located in north western, Kenya that straddles the Ethiopian border. With a surface area of about 6,750 km^2, it is Africa's fourth largest lake and the world's largest permanent desert lake. Numerous tribes depend increasingly upon the lake's fishery due to the unsustainable nature of their traditional livelihood of pastoralism in this arid region. Lake Turkana is also a haven for wildlife, supporting over 350 native and migratory bird species and the world's largest remaining population of the Nile crocodile. Owing to the faunal diversity and pale-anthropological importance of the region, also known as the "cradle of mankind", it has been named as a UNESCO World Heritage Site. It is widely known as the Jade Sea because of the remarkable, almost incandescent, color of its waters. The lake sustains 60 species of fish which are much sought-after by anglers, including tiger, cat and puffer fish, tilapia and Nile perch.

Since all things in an ecosystem are interdependent, the associations that exist between different organisms in the ecosystem influence the survival of organisms and the functioning of the ecosystem as a whole. Hence to understand the overall dynamics of the ecosystem, it is important to consider the impact of both environmental variations and multispecies interaction, which will have both ecological and evolutionary effects. Some of the ecological and evolutionary interactions are mutually beneficial, mutually detrimental or neutral.

Various ecological phenomena, massive number of predator-prey systems have been studied theoretically as well as numerically. Mainly mathematical models of predator-prey systems depend on the interaction of prey and predator population. In this study predation is the major mode of interaction and test case is where an organism feeds on another organism for survival. In predation, the predator feeds on another organism for their food while the prey is the organism to be fed on.

2. Literature Review

Modeling is a frequently evolving process meant for understanding the mathematical aspects of the problem and yielding non trivial biological insights, therefore it is constructed to give biologically meaningful and mathematically tractable population models, Ianneli and Pugliese [1]. Mathematical population models have been used to study the dynamics of prey- predator systems Lou [2] and Volterra [3] proposed the simple prey-predator model for the interactions now called the Lotka-Volterra model. Since then, many mathematical models have been constructed based on more realistic explicit and implicit biological assumptions.

Hence, according to this formulation predators never get satisfied as they will eat more and more prey around: for infinitely large prey abundances the predator feeding rate will also become infinite. The amount of prey eaten by a single predator per unit of time is referred to as the predator's functional response [2].

The basic Lotka-Volterra model and its' variant assume that the predators' functional response is a linear function of the prey abundance. This form is also known as a type I functional response, Dawes and Souza [4].According to Ianneli and Pugliese [1] organisms interact with other species and with the physical environment in many ways. These interactions sometimes include "negative feedbacks." An example of negative feedback is when an increase in the population of a prey species leads to an increase in the population of its predators (through increased reproduction), and this in turn feeds back to reduce the prey population through increased mortality from predation. Zhao and Kong [5] and Ahmed [6] developed models to determine the stability of prey-predator equilibrium as:

According to Turkana Fisheries Company, predation is common in the lake. This study seeks to investigate the balance of nature that exists between the *kingfishers* and pelagic fish in Lake Turkana. A mathematical model of ordinary non-linear differential equations is used to describe the relationship between the *kingfishers* and Pelagic fish and comparison graphs used to show how predation affects the population of the two species.

3. Model Formulation

In this predator-prey model the following assumptions are made:

i. The eco-system is closed i.e. there is no movement into and out of the lake.
ii. Only the Kingfisher (Predator) and the Pelagic Fish (Prey) are considered in the development of the model.
iii. It is assumed that the Kingfisher (Predator) depends only on the Pelagic Fish (Prey) as its' main source of food without which there will be significant decrease in the Common Kingfisher population in the absence of the pelagic fish (exponential decay).
iv. In the absence of the predator, increase in number of the prey will be observed and the growth will slow down once the pelagic fish population reaches the carrying capacity of the lake for the prey (logistic prey growth).
v. The prey (pelagic fish) has ample food supply.
vi. The environment does not change in favor of any of the two species e.g. climate change.

From the above assumptions let, K represent the population of the predator (Kingfisher) and F represent the population of the prey (pelagic fish).

Let the reproduction rate and death rate of the pelagic fish population (F) be r and μ respectively. The net rate of change of a population of the fish is equal to the rate-in of members (birth) minus the rate-out of members (death).

In the general situation, the increased competition as the population grows will result in a corresponding increase in death rate per individual and therefore it is assumed that the death rate per individual is directly proportional to the instantaneous population and that the birth rate per individual remains constant.

It is assumed that the Kingfisher converts the eaten pelagic fish into off-springs at a constant rate, β, though the conversion efficiency may not necessarily be the same as the quantity of consumed prey. Let α be the fish mortality rate due to attack by the predator.

For the Kingfisher population, K, assume that in the absence of the pelagic fish, the population of the common kingfisher will decrease and therefore the population of the common Kingfisher is directly proportional to the population of the pelagic fish.

The two differential equations governing the population of the two species are given by:

$$\frac{dF}{dt} = rF\left(1-\frac{F}{M}\right) - \beta FK \qquad (1)$$

$$\frac{dK}{dt} = \alpha FK - \mu K \qquad (2)$$

where M is the prey carrying capacity of the lake.

3.1. Steady States and Stability

Population equilibrium occurs in the model when neither of the population levels is changing, this implies that for steady states either $(F_1, K_1) = (0,0)$ and $(F_2, K_2) = \left(\frac{\mu}{\alpha}, \frac{r}{\beta}\left(1-\frac{\mu}{\alpha M}\right)\right)$.

To determine the stability of the steady states, the **Jacobian** of the system (1-2) is:

$$J(F,K) = \begin{pmatrix} r - \frac{2rF}{M} - \beta K & -\beta F \\ \alpha K & \alpha F - \mu \end{pmatrix} \qquad (3)$$

For the steady state, (F_1, K_1), the **Jacobian** simplifies to:

$$J(F_1, K_1) = \begin{pmatrix} r & 0 \\ 0 & -\mu \end{pmatrix} \qquad (4)$$

The Eigenvalues are $\lambda_1 < 0 < \lambda_2$ and therefore (0, 0) is a saddle point.

The Jacobian of the steady state (F_2, K_2) simplifies to:

$$J(F_2, K_2) = \begin{pmatrix} \frac{-r\mu}{\alpha M} & -\frac{\beta\mu}{\alpha} \\ \frac{r(\alpha M - \mu)}{\beta M} & 0 \end{pmatrix}. \qquad (5)$$

By determining the trace and the determinant as follows:

$Tr(J(F_2, K_2)) = \frac{-r\mu}{\alpha M} < 0$ and $Det(J(F_2, K_2)) = \frac{r\mu(\alpha M - \mu)}{\alpha M} > 0$ with all parameters and variables being greater than zero. In order for $K_2 > 0$ then $M > \frac{\mu}{\alpha}$. Since $Tr(J(F_2, K_2)) < 0$ and $Det(J(F_2, K_2)) > 0$ then (F_2, K_2) is stable.

3.2. Numerical Simulation

Using a Matlab solver to plot the solution and defining the constants as $r = 0.08, M = 5000, \mu = 0.02, \beta = 0.001, \alpha = 0.00025$ two scenarios are plotted. The first scenario Figure 1 shows the population of the fish against the population of the Kngfisher. The second case Figure 2 showing the two species populations with time. Taking the initial population of the pelagic fish to population of the Common Kingfisher as 0.1(Tonnes) to 0.12(Ten Thousands).

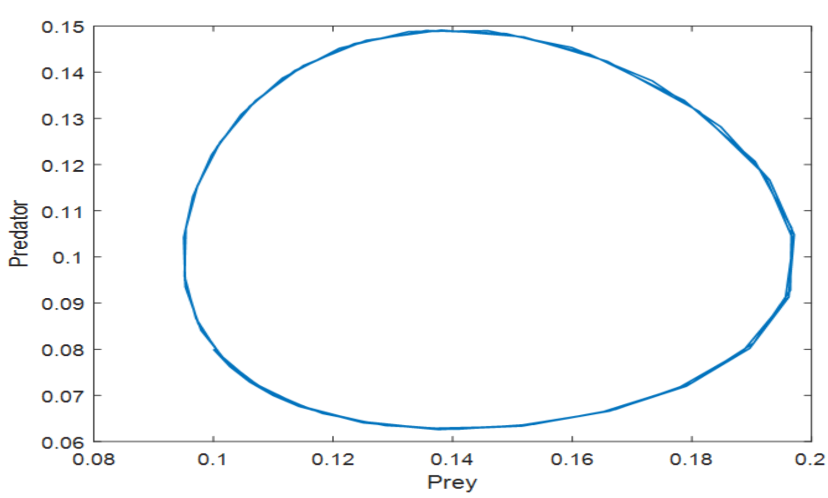

Figure 1. Graph predator against prey.

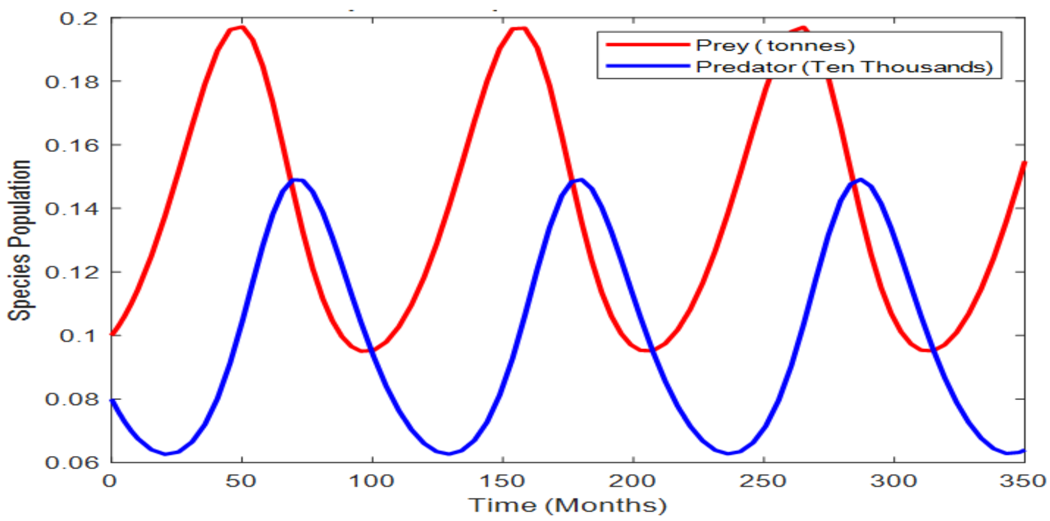

Figure 2. Species population against time.

Case1 (Figure 1)

Initially population of the pelagic fish is not enough to support the large number of the kingfisher and therefore the population of the kingfishers reduces drastically. With the reduced number of the predator, the prey increases up to an optimal number of 0.195 (tonnes). The population of the predator increases due to availability of prey and its optimal number of 0.15 (ten thousands) and the entire cycle commence afresh.

Case 2 (Figure 2)

Increase in the number of predators leads to decrease in the number of prey due to increased interactions between predators and prey, and therefore increased prey death. As the predator population decreases, the prey population begins to recover. Once the prey population has sufficiently recovered, the predator population once again increases.

3.3. Validation of Data

Table-1. Number of kingfishers and pelagic fish collected by fishermen in Lake Turkana from the year 1965 – 2011.

Year	**1965**	**1966**	**1967**	**1968**	**1969**	**1970**	**1971**	**1972**	**1973**	**1974**	**1975**	**1976**
Fish	3	6	2	3	61	64	75	87	112	122	30	63
Kingfisher	209	206	148	87	244	222	191	109	79	68	74	143

Year	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**
Fish	447	355	74	44	238	231	827	1068	1250	2218	2426	3018
Kingfisher	40	223	33	645	213	125	87	35	30	70	71	121

Year	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	**1995**	**1996**	**1998**	**1999**	**2000**
Fish	3196	3381	2092	1523	2556	2720	2695	2167	2266	2734	1990
Kingfisher	249	872	709	466	329	382	417	459	747	1209	567
Year	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2010**	**2011**	
Fish	2471	2279	2279	2279	2681	2640	2339	3090	2764	1968	
Kingfisher	882	829	799	983	710	696	546	522	431	329	

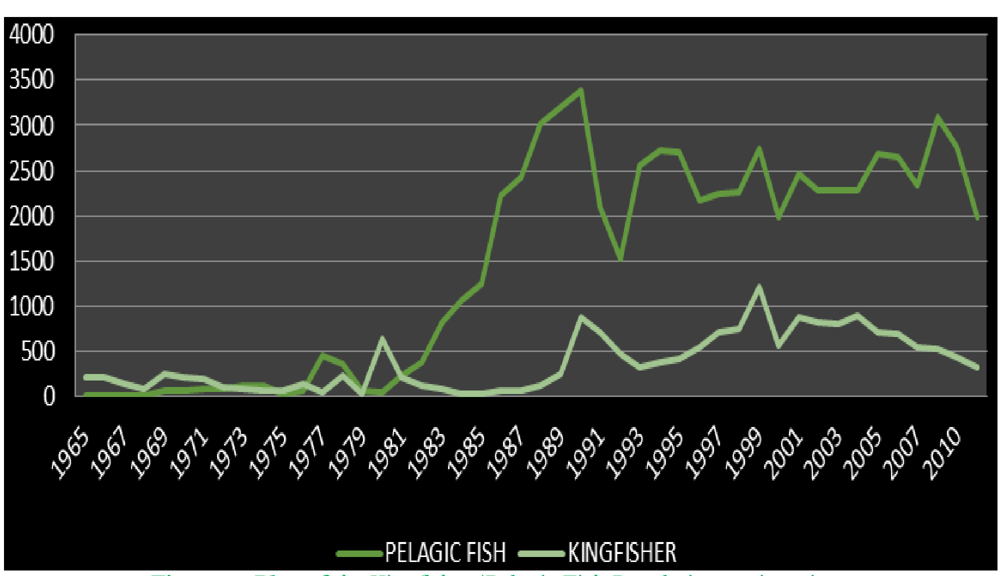

Figure-3. Plot of the Kingfisher/Palegic Fish Population against time.

From Figure 3, population of the Kingfisher increases, as that of Pelagic fish decreases, which soon leads to a decrease in the Kingfisher population as well and the trend repeat itself. The data presented in Table 1 showing the number of the Kingfisher/Palegic fish collected from the lake compares well with numerical results obtained from the model.

3. Conclusion

The main reason behind this research is the predation that is currently being witnessed in the lake. The data reveals that the relationship between the populations of the two species over time is well modelled by the predator-prey model. Analytical results demonstrates that the system has two equilibriums, namely the trivial equilibrium and the endemic equilibrium which are stable. Numerical simulation obtained from the model compares well with data obtained from Turkana Fisheries Company. It is fundamental to note that other animal species and human activities in the lake can cause imbalance in the ecosystem. Further research is recommended so as to ascertain the role of other species and human activities in the ecosystem.

References

[1] M. Ianneli and A. Pugliese, *An introduction to mathematical population dynamics*. Switzerland: Springer International Publishing, 2014.

[2] Y. Lou, *Global dynamics of a predator–prey model*. Guangzhou, 510631, PR China: South China Normal University, 2010.

[3] V. Volterra, *Variations and fluctuations in the numbers of coexisting animal species. In The Golden Age of Theoretical Ecology: 1923–1940*. Berlin, Heidelberg: Springer, 1978.

[4] J. Dawes and M. Souza, "A derivation of Holling's type I, II and III functional responses in predator–prey systems," *Journal of Theoretical Biology*, vol. 327, pp. 11-22, 2013. Available at: https://doi.org/10.1016/j.jtbi.2013.02.017.

[5] X. Zhao and L. Kong, "Stability analysis of a predator-prey system," presented at the 2012 Fourth International Conference on Computational and Information Sciences, Chongqing, 2012.

[6] B. A. Ahmed, "Stability analysis of a predator-prey," *Model Universal Journal of Applied Mathematics & Computation*, vol. 5, pp. 20-24, 2017.