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Analysis and control of the permanent magnet synchronous motor model

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Abstract

The permanent magnet synchronous motor (PMSM) is rapidly becoming a cornerstone of diesel–electric ship propulsion. The dynamics of the PMSM are highly nonlinear and require thorough understanding to enable efficient operation. In this work, bifurcation analysis and multi-objective nonlinear model predictive control (NMPC) are performed on a PMSM model. The PMSM is frequently used for diesel–electric ship propulsion. Bifurcation analysis is a powerful mathematical tool used to address the nonlinear dynamics of such processes. Several factors must be considered, and multiple objectives must be met simultaneously. MATLAB program MATCONT was employed to perform the bifurcation analysis. The MNLMPC calculations were carried out using the optimization language PYOMO, in conjunction with advanced global optimization solvers IPOPT and BARON. The bifurcation analysis revealed the existence of Hopf bifurcation points and a limit point. The MNLMC converged on the Utopian solution. The Hopf bifurcation point, which causes an undesirable limit cycle, is eliminated using an activation factor involving the tanh function. The limit point, which can lead to multiple steady-state solutions, is advantageous because it allows the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point, representing the optimal solution in the model.

Keywords: Bifurcation, Control, Diesel-electric, Optimization, Permanent magnet Synchronous motor.

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Contents

Contribution of this paper to the literature
A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for a PMSM model is the main contribution of this paper. The integration of MNLMPC and bifurcation analysis for a PMSM model is the novelty of this work.

1. Introduction

A permanent magnet synchronous motor, or PMSM, has increasingly become one of the most important components in advanced propulsion systems, especially in the maritime sector where diesel–electric configurations are widely applied. The diesel–electric ship propulsion system itself is a hybrid between traditional mechanical propulsion and full electric drive, in which diesel engines do not directly power the propeller shaft but instead drive generators that produce electrical energy. This electricity is then distributed to propulsion motors and other onboard loads, providing operational flexibility, efficiency, and control. The use of PMSMs in this context represents a major step forward in naval architecture and marine engineering because these machines combine the efficiency and controllability of synchronous machines with the compactness and reliability of permanent magnets. In practice, PMSMs are now being considered as replacements or alternatives to induction motors and wound-field synchronous machines in many vessel classes, ranging from commercial cargo ships to naval platforms and specialized offshore support vessels.

The Permanent Magnet Synchronous Motor (PMSM) is a rotating electrical machine that operates synchronously with the supply frequency. Unlike induction motors, which depend on slip to induce rotor currents, the PMSM employs high-energy permanent magnets embedded or mounted on the rotor surface to establish the rotor magnetic field. This design eliminates the need for rotor excitation windings, brushes, or slip rings, thereby reducing losses, increasing efficiency, and minimizing maintenance requirements. In the context of ship propulsion, these features are particularly advantageous because marine environments demand high reliability, reduced downtime, and low operating costs. The synchronous operation also offers the benefit of precise speed control and high torque density, which are essential for maneuvering large vessels in port or navigating through restricted waterways.

In diesel–electric ship propulsion, the role of PMSMs is intimately connected to the overall energy chain. Diesel generators convert fuel energy into electrical energy, which is distributed through switchboards and power electronics to the propulsion motor. The PMSM, when supplied with controlled three-phase alternating current through a voltage source inverter, produces a rotating magnetic field in the stator that interacts with the rotor's permanent magnets to develop torque. The absence of slip ensures that torque production is immediate and directly proportional to current, making the PMSM highly responsive to control commands. This responsiveness is advantageous in marine applications, where propulsion demands can change rapidly due to varying sea states, maneuvers, and dynamic positioning requirements.

Another major advantage of PMSMs in diesel–electric propulsion is their high efficiency across a broad operating range. Because the rotor magnetic field is provided by permanent magnets, no additional electrical power is required for field excitation, reducing copper losses. Furthermore, advanced rare-earth permanent magnets such as neodymium–iron–boron allow for very high magnetic flux densities, increasing the torque-to-weight ratio of the machine. In a ship, where space and weight are critical constraints, the ability to deliver large torque in a compact footprint means that PMSMs can be integrated into podded propulsors or other constrained propulsion modules. This results in quieter operation, better hydrodynamic efficiency, and improved hull form flexibility for designers. The efficiency gains also translate directly into lower fuel consumption and reduced greenhouse gas emissions, aligning with increasingly strict international regulations on maritime emissions.

However, the integration of PMSMs into diesel–electric propulsion systems requires careful consideration of power electronics and control strategies. Since PMSMs cannot be directly connected to the generator bus due to their synchronous nature, they must be operated through inverters that provide variable frequency and voltage to control speed and torque. Field-oriented control (FOC) and direct torque control (DTC) strategies are typically used, relying on sophisticated algorithms and sensors to manage stator currents and rotor position. The complexity of these systems increases the initial investment and demands a high level of technical expertise for operation and maintenance. Yet, the benefits in terms of dynamic performance, energy savings, and reduced lifecycle costs often justify the investment, especially for high-value vessels operating on long duty cycles.

Noise and vibration reduction is another area where PMSMs have a decisive advantage in marine applications. Traditional propulsion systems, particularly those using direct diesel-mechanical drives, suffer from torsional vibrations, noise, and resonance issues that propagate through the ship's structure. PMSMs, when properly controlled, provide smooth torque delivery and reduced mechanical vibration, which not only improves passenger comfort in cruise ships and ferries but also reduces acoustic signatures for naval vessels engaged in stealth operations. Additionally, lower vibration and noise reduce fatigue on structural components and onboard equipment, extending the vessel's operational life.

Reliability and maintainability are crucial in marine propulsion, where failure at sea can have catastrophic consequences. The elimination of brushes, slip rings, and rotor windings in PMSMs reduces the number of components prone to wear and failure. Permanent magnets, once properly manufactured and encapsulated, have very long lifespans with negligible degradation under normal operating conditions. Nonetheless, designers must account for potential demagnetization risks due to excessive temperatures or fault conditions. Advanced cooling systems, robust thermal design, and appropriate protection schemes are therefore essential for marine PMSMs. In practice, liquid cooling systems are often integrated into the stator to maintain stable operating temperatures, ensuring both performance and longevity.

Cost considerations remain one of the limiting factors in the widespread adoption of PMSMs for diesel–electric propulsion. The high cost of rare-earth permanent magnets, particularly neodymium, contributes significantly to the overall price of these machines. Moreover, supply chain uncertainties related to rare-earth materials raise concerns about long-term availability and price stability. Despite this, when lifecycle costs are considered, PMSMs often prove competitive because of their lower operating costs, reduced fuel consumption, and minimal maintenance requirements. For shipowners and operators facing stringent emission regulations and rising fuel prices, these long-term benefits are becoming increasingly attractive.

The adoption of PMSMs in diesel–electric propulsion also aligns with the broader shift toward hybrid and fully electric ships. As battery technologies advance and renewable energy sources such as fuel cells are integrated into ship power systems, PMSMs provide an optimal match due to their efficiency, controllability, and scalability. In hybrid vessels, PMSMs can seamlessly switch between diesel-generated power and stored energy sources, providing flexible and resilient propulsion. In fully electric ships, PMSMs maximize the utility of limited onboard energy storage, extending range and endurance. This compatibility with future propulsion architectures ensures that PMSMs will play a critical role in the decarbonization of maritime transport.

Case studies have already demonstrated the effectiveness of PMSM-based propulsion systems. Many modern icebreakers, offshore supply vessels, and cruise ships have adopted podded propulsion units driven by PMSMs, benefiting from their compactness, efficiency, and maneuverability. The Azipod system, for example, uses PMSMs to provide omnidirectional thrust, significantly improving vessel handling and fuel efficiency. In naval applications, PMSMs are favored for their low acoustic signatures and high torque density, enabling advanced warship designs with enhanced stealth and performance.

In conclusion, the permanent magnet synchronous motor is rapidly becoming a cornerstone of diesel–electric ship propulsion. By combining high efficiency, compactness, reliability, and controllability, PMSMs address many of the limitations of traditional propulsion motors and offer significant advantages for modern vessels. Although challenges remain in terms of cost, rare-earth material dependence, and control complexity, the long-term benefits in operational efficiency, emissions reduction, and adaptability to hybrid and electric propulsion architectures make PMSMs a compelling choice for the future of maritime transport. As the shipping industry continues to evolve under the pressures of environmental regulation, fuel economy, and technological innovation, the role of PMSMs in diesel–electric ship propulsion is likely to expand, solidifying their place as a critical enabler of sustainable and efficient marine propulsion.

Jianbo et al. [1] studied the direct active and reactive power control of PMSM. Ho et al. [2] designed a PMSM motor drive with active power factor correction. Parvathy et al. [3] applied quadratic linearization to control a permanent magnet synchronous motor. Cimini et al. [4] studied PMSM control with power factor correction. Lei et al. [5] investigated the unit power factor control of PMSM fed by an indirect matrix converter. Do et al. [6] developed a suboptimal control scheme design for interior permanent-magnet Synchronous Motors. Hansen and Wendt [7] discussed the state of the art in commercial electric ship propulsion integrated power systems. Grljušić et al. [8] calculated the efficiencies of a ship power plant operating with waste heat recovery through combined heat and power production. Veksler et al. [9] investigated the thrust allocation with dynamic power consumption modulation for diesel-electric ships. Ludwig and Möckel [10] discussed the operation method for a grid-powered PMSM with open-end winding in a dual-inverter topology for power factor maximization. Kozak [11] investigated the concept of a ship's power plant system with varying rotational speed gensets. Vimal and Sojan [12] studied the vector-controlled PMSM drive with power factor correction using a zeta converter. Choi et al. [13] discussed the variable speed control of a diesel engine-generator using sliding mode control.

Gokulapriya and Pradeep [14] developed a shunt-based active power factor correction circuit for a direct torque-controlled PMSM drive. Mrzljak et al. [15] provided the numerical analysis of efficiencies and important operating parameters for a marine slow-speed two-stroke diesel engine. Zhou et al. [16] developed a coordinated power control of variable-speed diesel generators and lithium batteries on a hybrid electric boat. Aghili [17] discussed optimal feedback linearization control of interior permanent magnet synchronous motors subject to time-varying operation conditions, minimizing power loss. German-Galkin and Tarnapowicz [18] investigated energy optimization of a ship's shaft generator with a permanent magnet synchronous generator. German-Galkin and Tarnapowicz [19] researched optimization of the electric car's drive system with PMSM. Zaccone et al. [20] optimized a diesel-electric ship propulsion and power generation system using a genetic algorithm. Acanfora et al. [21] studied load levelling through a storage system for hybrid diesel electric ship propulsion in irregular wave conditions. Rezkallah et al. [22] developed a coordinated control strategy for a hybrid off-grid system based on a variable-speed diesel generator. Zwierzewicz et al. [23] performed optimal control studies of the diesel–electric propulsion in a ship with PMSM. In this work, bifurcation analysis and multiobjective nonlinear model predictive control are performed on a PMSM model [23]. The paper is organized as follows. First, the model equations are presented, followed by a discussion of the numerical techniques involving bifurcation analysis and multiobjective nonlinear model predictive control (MNLMPC). The results and discussion are then presented, followed by the conclusions.

2. Model Equations

In this model, u1 is the inverter output voltage on the d-axis, u2 is the inverter output voltage on the q-axis, x1 is the stator current on the d-axis, and x2 is the stator current on the q-axis. Additionally, x3 is the angular speed of the machine's rotor. The parameter jpar represents inertia, ppar is the number of machine's pole pairs, y0 is the rated rotor flux linkage, tl is the load torque, and r1 represents the stator winding phase resistance.

The model equations are,

$$\frac{d(xv1)}{dt} = -r1(\frac{xv1}{l1}) + ppar(xv3)xv2 + \frac{u1}{l1}$$
$$\frac{d(xv2)}{dt} = -ppar(xv3)xv1 - r1(\frac{xv2}{1}) - ppar(y0)\frac{xv3}{l1} + \frac{u2}{l1} \qquad (1)$$
$$\frac{d(xv3)}{dt} = 1.5(ppar)y0(\frac{xv2}{jpar}) - \frac{tl}{jpar}$$

The base parameter values are,

r1=0.821; l1=1.573; ppar=26; y0=5.8264; jpar=6; tl=848.826; u1=0; u2=4.

3. Bifurcation Analysis

For bifurcation analysis, which deals with multiple steady states (that occur because of branch and limit points) and limit cycles (caused by Hopf bifurcation points), the program MATCONT [24, 25] is used in a system of equations.

$$\frac{dx}{dt} = f(x,\alpha) \qquad (2)$$

$x \in R^n$ and α is the bifurcation parameter.

The tangent is the n+1-dimensional vector w that satisfies.

$$Aw = 0 \qquad (3)$$

where

$$A = [\partial f/\partial x \quad |\partial f/\partial \alpha] \qquad (4)$$

And $\partial f/\partial x$ is the Jacobian matrix. The Jacobian matrix $J = [\partial f/\partial x]$ must be singular for both limit and branch points.

There is only one tangent at the point of singularity for a limit point. At this singular point, there is a single non-zero vector, y, where Jy=0. This vector is of dimension n. Since there is only one tangent the vector $y = (y_1, y_2, y_3, y_4, \ldots y_n)$ must align with $\hat{w} = (w_1, w_2, w_3, w_4, \ldots w_n)$. Since

$$J\hat{w} = Aw = 0 \qquad (5)$$

The n+1 th component of the tangent vector $w_{n+1} = 0$.

There are two tangents (z and w) at the singular point for a branch point. This implies that.

$$Az = 0$$
$$Aw = 0 \qquad (6)$$

Consider a vector v that is orthogonal to say w. v can be expressed as a linear combination of z and w ($v = \alpha z + \beta w$). Since $Az = Aw = 0$; $Av = 0$ and since w and v are orthogonal.

$w^T v = 0$. Hence $Bv = \begin{bmatrix} A \\ w^T \end{bmatrix} v = 0$ which implies that B is singular.

Hence, the matrix $B = \begin{bmatrix} A \\ w^T \end{bmatrix}$ is singular at a branch point.

For a Hopf bifurcation point, the bialternate product.

$$det(2f_x(x,\alpha)@Jn) = 0 \qquad (7)$$

where Jn is the n-square identity matrix? Hopf bifurcations cause limit cycles and should be eliminated because limit cycles make optimization and control tasks very difficult. More details can be found in Kuznetsov [26], Kuznetsov [27], and Govaerts [28].

Hopf bifurcations cause limit cycles, which cause equipment damage and make control tasks difficult and also result in less beneficial products. Sridhar [29] showed that the tanh activation function (where the manipulated variable is modified to $(v \tanh v/\varepsilon)$ eliminates Hopf bifurcation points by increasing the oscillation time period in the limit cycle.

4. Multi-objective Nonlinear Model Predictive Control (MNLMPC)

The MNLMPC method developed by Flores-Tlacuahuac et al. [30] was used. For a problem where the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ subject to the constraints.

$$\frac{dx}{dt} = F(x,u) \qquad (8)$$

t_f represents the final time value and u the control variable. The individual objective optimal control problem is solved by optimizing each of the variables $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$. The optimization of $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ will result in the values q_j^* Then the multiobjective optimal control problem is solved.

$$min(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$$
$$subject\ to\ \frac{dx}{dt} = F(x,u); \qquad (9)$$

This will provide the value of u at each time step. The first obtained control value of u is implemented and this procedure is repeated until the implemented and the first obtained control values are the same or where ($\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^* = 0$ for all j. Utopia point) is obtained.

The optimization program PYOMO [31] is used in conjunction with IPOPT [32] and BARON [33].

The steps of the algorithm involve.

1. Optimizing $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i)$ to get q_j^*.
2. Minimizing $(\sum_{j=1}^{n}(\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^*))^2$ to get the control values at various times.
3. Implement the first obtained control values and repeat until there is an insignificant difference between the implemented and the first obtained value of the control variables or if the Utopia point is achieved. The Utopia point is when $\sum_{t_{i=0}}^{t_i=t_f} q_j(t_i) - q_j^* = 0$ for all j.

Sridhar [34] proved that when bifurcation analysis revealed the presence of limit and branch points, the MNLMPC calculations converged to the Utopia solution.

5. Results and Discussion

A permanent magnet synchronous motor is one of the most widely used types of electric machines in modern technology, particularly in applications that demand high efficiency, precise control, and reliability such as electric vehicles, renewable energy systems, industrial automation, robotics, and increasingly ship propulsion and aerospace. The defining characteristic of a PMSM is its use of permanent magnets on the rotor, which eliminates the need for external excitation and provides a strong magnetic field that contributes to high power density and efficiency. Like all dynamical systems, however, PMSMs are governed by nonlinear differential equations that describe the interaction between electrical and mechanical subsystems, including rotor dynamics, stator currents, and electromagnetic torque. While under normal operating conditions these dynamics can be well controlled using modern strategies such as vector control or direct torque control, PMSMs remain susceptible to undesirable nonlinear phenomena. One of the most critical of these is the occurrence of Hopf bifurcations, which can have significant negative consequences for performance, stability, and safety.

A Hopf bifurcation occurs in a nonlinear system when a pair of complex conjugate eigenvalues of the system Jacobian cross the imaginary axis as a parameter changes, leading to a transition from a stable equilibrium to sustained oscillatory behavior, or vice versa. In the context of PMSMs, such a bifurcation often manifests as self-sustained oscillations in rotor speed, torque, or currents that arise even in the absence of external periodic forcing. These oscillations correspond to the emergence of limit cycles, meaning that the motor state no longer converges smoothly to the desired equilibrium point but instead exhibits persistent fluctuations. This behavior is fundamentally problematic because a PMSM is typically expected to deliver smooth torque, maintain precise speed regulation, and operate with minimal acoustic noise and vibration. The very occurrence of a Hopf bifurcation undermines these expectations and introduces a host of challenges.

One of the main reasons Hopf bifurcations are detrimental to PMSM operation is that they compromise stability in a way that is difficult to manage through standard control methods. Linear control strategies such as proportional–integral–derivative controllers or linear quadratic regulators assume small perturbations around a stable operating point and rely on the system being locally asymptotically stable. When a Hopf bifurcation occurs, however, the system ceases to have a stable equilibrium, and instead trajectories settle onto a periodic orbit. This renders traditional linear feedback methods ineffective, since the controller may attempt to correct for deviations that are no longer deviations but part of the new dynamical regime. The result can be degraded control performance, excessive controller effort, or even exacerbation of the oscillations. For high-precision applications, such as robotic manipulators or machine tool drives where PMSMs are favored, this loss of stability is unacceptable.

Another important issue is that oscillations caused by Hopf bifurcations lead directly to torque ripple, current distortion, and speed fluctuations. These effects reduce the efficiency of the motor, increase power losses, and generate heat. In high-power PMSMs, such as those used in electric vehicles or ship propulsion, sustained oscillations can stress the power electronics that drive the motor, causing higher switching losses, voltage overshoot, and increased harmonic content. This not only shortens the lifespan of the motor and its associated hardware but also reduces energy efficiency, which is particularly undesirable in applications where energy conservation is critical. Moreover, oscillations in torque translate into mechanical vibrations, which in turn lead to wear and tear of mechanical components, higher acoustic noise, and potential resonance with structural modes. Over long periods, this can accelerate mechanical fatigue and increase maintenance costs.

From a safety perspective, Hopf bifurcations represent a hidden risk because they are parameter-dependent and may arise unexpectedly as system conditions vary. For example, PMSMs are often controlled over wide ranges of operating speeds and loads, with control gains or feedback parameters being adjusted adaptively. At certain critical values, these changes may inadvertently push the system through a bifurcation point, triggering oscillations that were not previously present. This is particularly hazardous in safety-critical applications like electric aircraft actuation or naval propulsion, where unexpected oscillations in motor dynamics can translate to instability at the system level. The difficulty lies in the fact that Hopf bifurcations often occur gradually as a parameter passes a critical threshold, but the consequences manifest suddenly once oscillations grow to appreciable amplitude. This makes prediction, detection, and mitigation challenging in real time.

In addition, Hopf bifurcations complicate the design and tuning of controllers for PMSMs. To ensure stability, engineers must account not only for linear stability margins but also for nonlinear effects that can lead to oscillatory instabilities. This requires more sophisticated analysis methods such as bifurcation theory, Lyapunov-based nonlinear stability analysis, or numerical continuation methods to map out the regions of parameter space that are safe. Such analysis adds significant complexity to the design process and often requires conservative choices that limit performance. For instance, a controller might need to be detuned to avoid oscillations, which reduces dynamic responsiveness and torque accuracy. The very need to account for Hopf bifurcations thus increases both development time and system cost.

From an operational point of view, Hopf bifurcations also reduce robustness to disturbances. A PMSM operating near a bifurcation point may be stable under nominal conditions but highly sensitive to noise, parameter uncertainty, or load variations. A small disturbance can trigger the system into oscillatory behavior, which then persists indefinitely due to the limit cycle nature of the dynamics. This sensitivity undermines the reliability of PMSMs in real-world environments where disturbances are unavoidable, whether due to variable loads, sensor noise, or thermal fluctuations. In transportation systems, where PMSMs are increasingly used, such sensitivity could lead to performance degradation at critical moments, for example during acceleration or maneuvering.

Furthermore, Hopf bifurcations have implications for the broader system in which the PMSM is embedded. In electric vehicles, oscillations in motor torque can propagate to the drivetrain, reducing ride comfort and possibly interacting with vehicle dynamics. In grid-connected applications, oscillations in motor currents can contribute to power quality issues, including harmonics and voltage instability. These system-level consequences mean that a Hopf bifurcation in a PMSM is not merely a local motor issue but a source of instability that can cascade into other subsystems.

Finally, Hopf bifurcations present a challenge for diagnostics and monitoring. Traditional condition monitoring systems for PMSMs are designed to detect faults such as demagnetization, bearing wear, or insulation failure. Oscillatory instabilities induced by bifurcations do not fit neatly into these categories and may be misinterpreted as sensor noise or minor transients. Consequently, operators may not realize that the system has entered an unstable regime until the oscillations grow large enough to cause visible performance degradation. By then, efficiency losses, component stresses, and safety risks may already be significant. Designing diagnostic systems capable of detecting bifurcation-induced oscillations requires advanced signal analysis and real-time nonlinear modeling, which are not yet standard in most industrial applications.

Hopf bifurcations are detrimental to PMSMs because they destabilize the fundamental operating principle of smooth, reliable torque and speed control. They transform stable equilibria into persistent oscillations, leading to efficiency loss, increased wear and noise, safety hazards, and greater system complexity. They also complicate control design, reduce robustness, and create risks that are difficult to diagnose in real time. While bifurcation theory provides valuable insights into how and when such instabilities arise, the practical reality is that engineers must go to great lengths to design PMSMs and their controllers to avoid them. In a technological landscape where PMSMs are increasingly critical for sustainable energy and transportation, ensuring that Hopf bifurcations are suppressed or avoided entirely is essential for achieving reliable and efficient operation. Thus, Hopf bifurcations are not merely a theoretical curiosity in nonlinear dynamics but a very real engineering problem that must be recognized, analyzed, and prevented in the design and operation of PMSM-driven systems.

When u1 was the bifurcation parameter, a Hopf bifurcation point was found at (xv1, xv2, xv3, u1) values of (-1.643861, 3.735543, 0.011075, -3.041582) (curve AB in Figure 1a). The limit cycle that occurs because of this Hopf bifurcation is shown in Figure 1b. When u1 is modified to u1tanh(u1), the Hopf bifurcation disappears (curve CD in Figure 1a), validating the analysis of Sridhar [29]. When r1 is the bifurcation parameter, a limit point occurs at (xv1, xv2, xv3, r1) of (-1.852002 3.735543 -0.017211 1.419769) (Figure 1c).

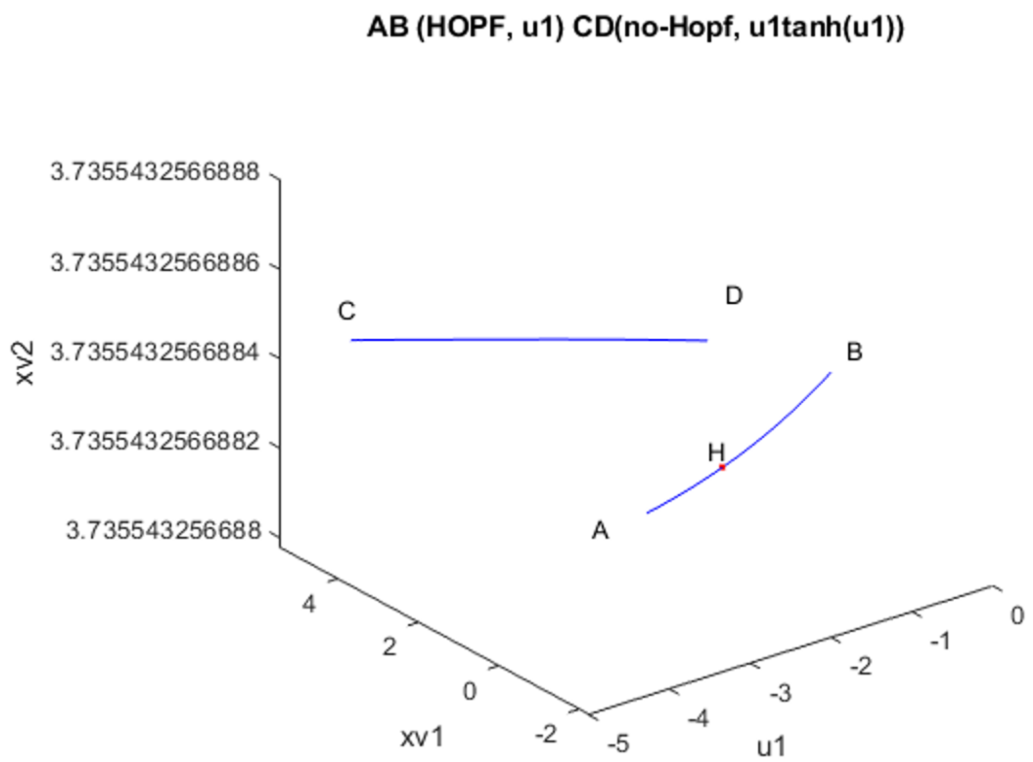

Figure 1a. Bifurcation diagram (u1 is bifurcation parameter).

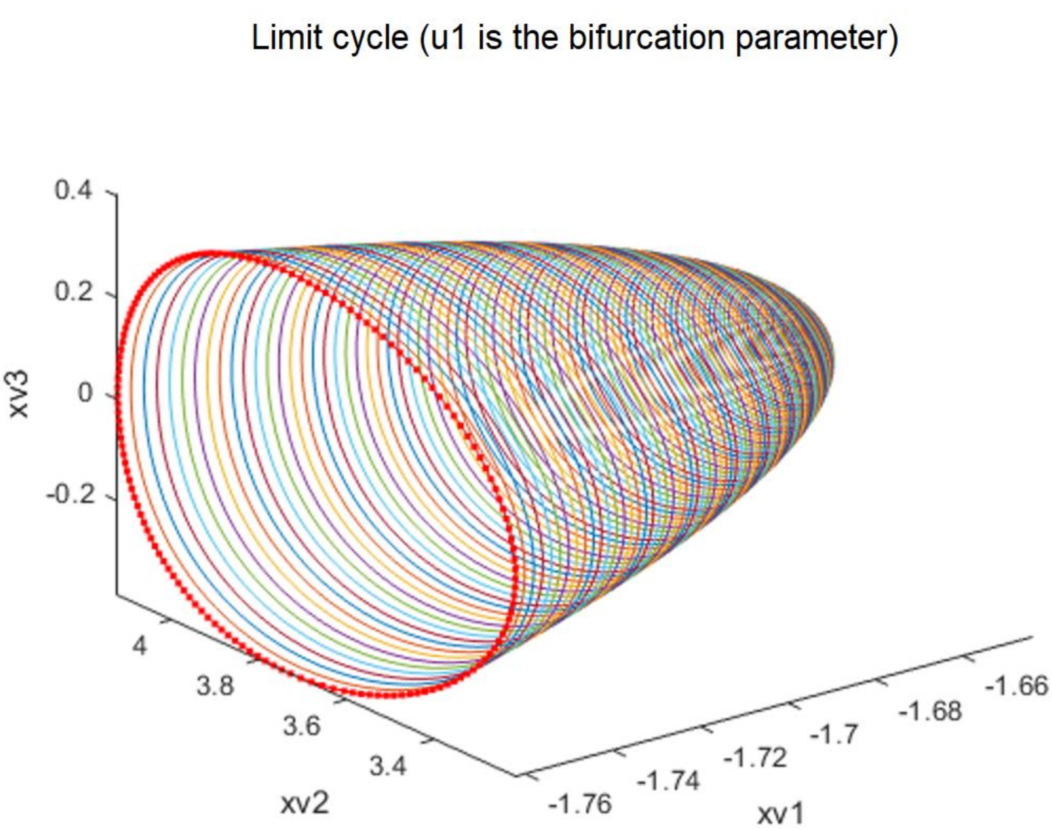

Figure 1b. Limit cycle when u1 is the bifurcation parameter.

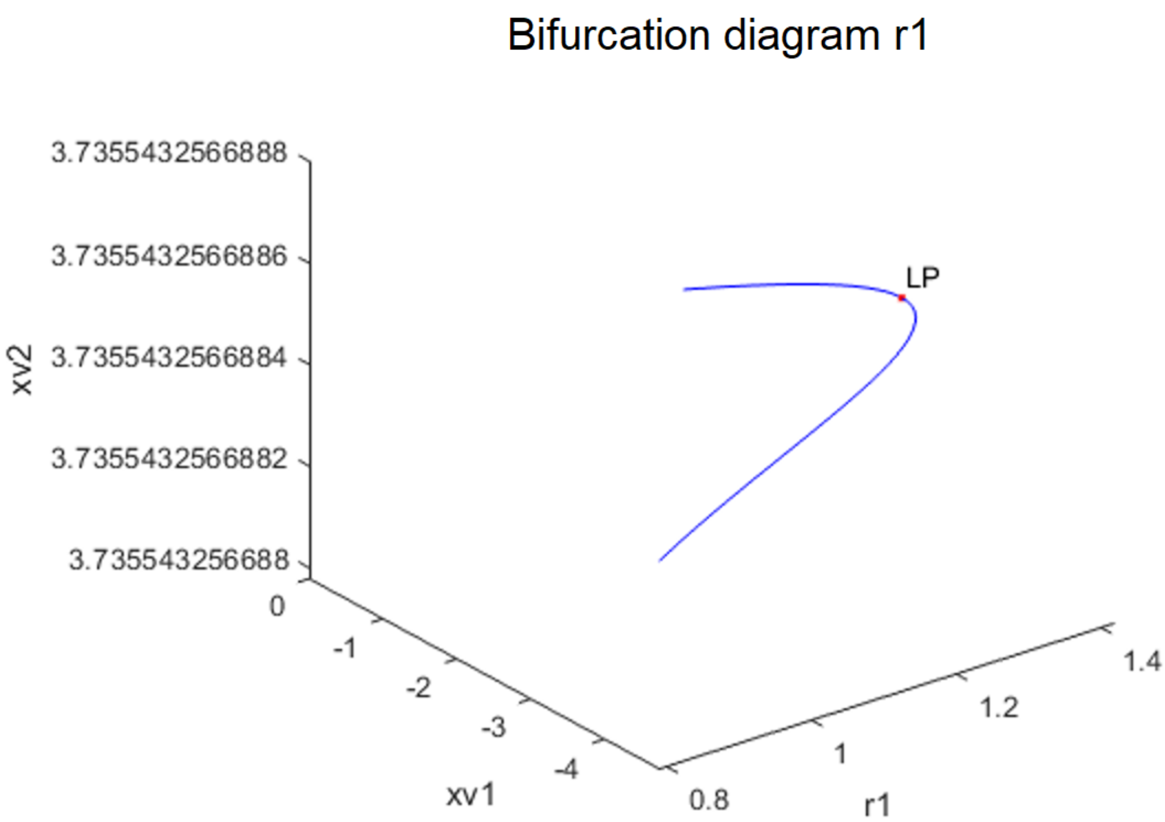

Figure 1c. Bifurcation diagram r1 is the bifurcation parameter.

For the MNLMPC, r1 is the control parameter, and $\sum_{t_{i=0}}^{t_i=t_f} u1(t_i), \sum_{t_{i=0}}^{t_i=t_f} u2(t_i)$ were maximized individually, and each of them led to a value of 20. The overall optimal control problem will involve the minimization of $(\sum_{t_{i=0}}^{t_i=t_f} u11(t_i) - 20)^2 + (\sum_{t_{i=0}}^{t_i=t_f} u2(t_i) - 20)^2$ was minimized subject to the equations governing the model. This led to a value of zero (the Utopia point). The MNLMPC values of the control variable, r1, are 2.305. The limit point causes the MNLMPC calculations to attain the Utopia solution, validating the analysis of Sridhar [34]. Figures 2a and 2b show the MNLMPC profiles.

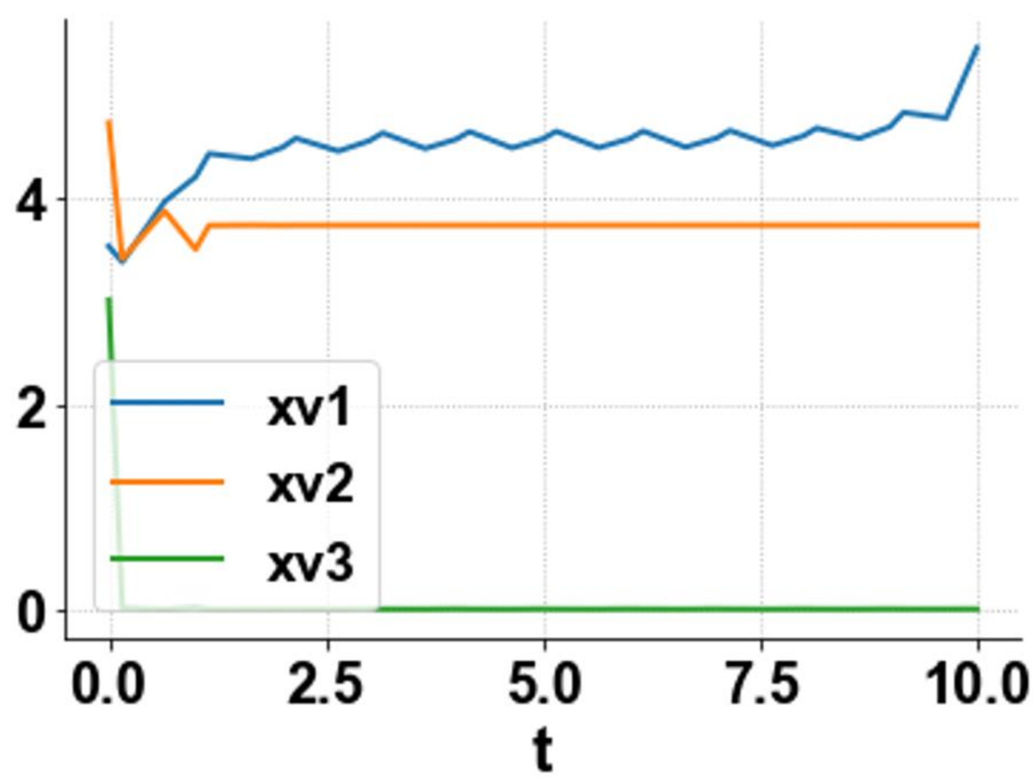

Figure 2a. MNLMPC xv1, xv2, xv3.

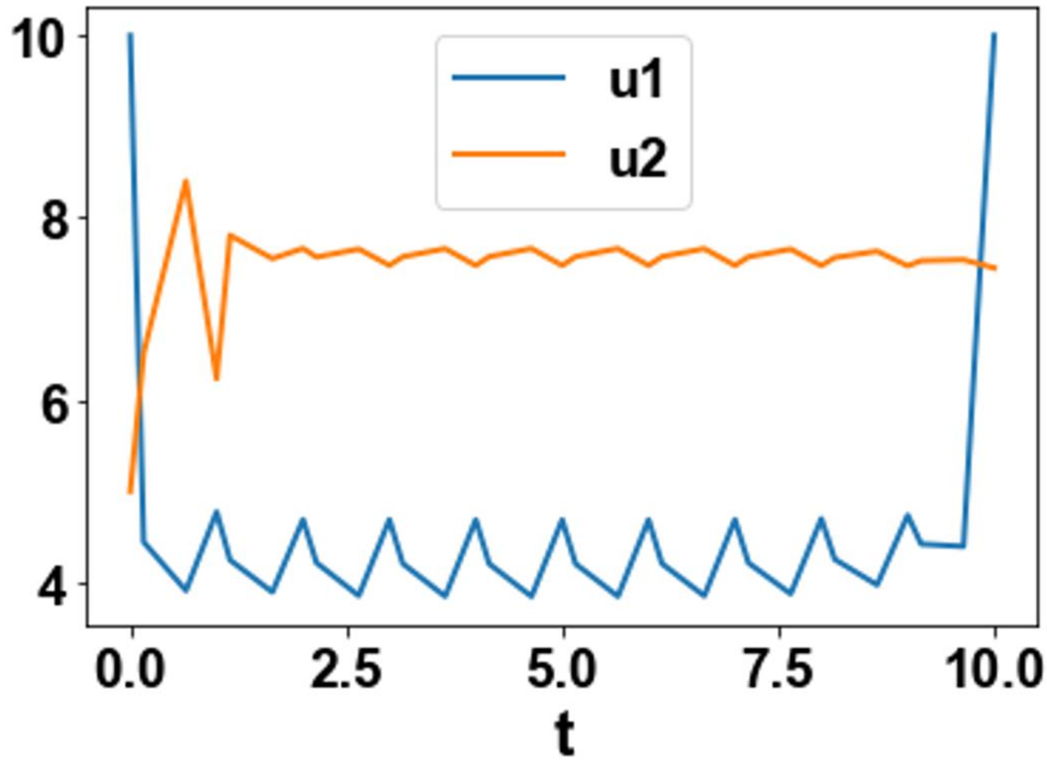

Figure 2b. MNLMPC u1, u2.

6. Conclusions

Bifurcation analysis and multiobjective nonlinear control (MNLMPC) studies on a permanent magnet synchronous motor (PMSM) model. The permanent magnet synchronous motor (PMSM) is used for diesel–electric ship propulsion. The bifurcation analysis revealed the existence of Hopf bifurcation points, limit points, and branch points. The Hopf bifurcation point, which causes an unwanted limit cycle, is eliminated using an activation factor involving the tanh function. The limit point (which causes multiple steady-state solutions from a singular point) is very beneficial because it enables the Multiobjective nonlinear model predictive control calculations to converge to the Utopia point (the best possible solution) in the models. A combination of bifurcation analysis and Multiobjective Nonlinear Model Predictive Control (MNLMPC) for a PMSM model is the main contribution of this paper.

References

[1] C. Jianbo, H. Yuwen, H. Wenxin, L. Yong, Y. Jianfei, and W. Mingjin, "Direct active and reactive power control of PMSM," presented at the 2009 IEEE 6th International Power Electronics and Motion Control Conference, IEEE, 2009.

[2] T.-Y. Ho, M.-S. Chen, L.-Y. Chen, and L.-H. Yang, "The design of a PMSM motor drive with active power factor correction," presented at the 2011 2nd International Conference on Artificial Intelligence, Management Science and Electronic Commerce (AIMSEC), IEEE, 2011.

[3] A. Parvathy, R. Devanathan, and V. Kamaraj, "Application of quadratic linearization to control of Permanent Magnet synchronous motor," presented at the 2011 1st International Conference on Electrical Energy Systems, IEEE, 2011.

[4] G. Cimini, G. Ippoliti, G. Orlando, and M. Pirro, "PMSM control with power factor correction: Rapid prototyping scenario," presented at the 4th International Conference on Power Engineering, Energy and Electrical Drives, IEEE, 2013.

[5] J. Lei, B. Zhou, and J. Bian, "Unit power factor control of PMSM fed by indirect matrix converter," presented at the 2014 17th International Conference on Electrical Machines and Systems (ICEMS), IEEE, 2014.

[6] T. D. Do, S. Kwak, H. H. Choi, and J.-W. Jung, "Suboptimal control scheme design for interior permanent-magnet synchronous motors: An SDRE-based approach," *IEEE Transactions on Power Electronics*, vol. 29, no. 6, pp. 3020-3031, 2013. https://doi.org/10.1109/TPEL.2013.2272582

[7] J. F. Hansen and F. Wendt, "History and state of the art in commercial electric ship propulsion, integrated power systems, and future trends," *Proceedings of the IEEE*, vol. 103, no. 12, pp. 2229-2242, 2015. https://doi.org/10.1109/JPROC.2015.2458990

[8] M. Grljušić, V. Medica, and G. Radica, "Calculation of efficiencies of a ship power plant operating with waste heat recovery through combined heat and power production," *Energies*, vol. 8, no. 5, pp. 4273-4299, 2015. https://doi.org/10.3390/en8054273

[9] A. Veksler, T. A. Johansen, R. Skjetne, and E. Mathiesen, "Thrust allocation with dynamic power consumption modulation for diesel-electric ships," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 2, pp. 578-593, 2015. https://doi.org/10.1109/TCST.2015.2446940

[10] F. Ludwig and A. Möckel, "Operation method for AC grid powered PMSM with open-end winding in dual-inverter topology for power factor maximization," presented at the 8th IET International Conference on Power Electronics, Machines and Drives (PEMD 2016), IET, 2016.

[11] M. Kozak, "New concept of ship's power plant system with varying rotational speed gensets," in *Proceedings of the 58th International Conference of Machine Design Departments, Prague, Czech Republic*, 2017, pp. 8-10.

[12] M. Vimal and V. Sojan, "Vector controlled PMSM drive with power factor correction using zeta converter," presented at the 2017 International Conference on Energy, Communication, Data Analytics and Soft Computing (ICECDS), IEEE, 2017.

[13] I.-C. Choi, Y.-C. Jeung, and D.-C. Lee, "Variable speed control of diesel engine-generator using sliding mode control," presented at the 2017 IEEE Transportation Electrification Conference and Expo, Asia-Pacific (ITEC Asia-Pacific), IEEE, 2017.

[14] R. Gokulapriya and J. Pradeep, "Shunt based active power factor correction circuit for direct torque controlled PMSM drive," in *Proceedings of the 2017 Third International Conference on Science Technology Engineering & Management (ICONSTEM), Chennai, India. IEEE*, 2017, pp. 517–521.

[15] V. Mrzljak, B. Žarković, and J. Prpić-Oršić, "Marine slow speed two-stroke diesel engine—Numerical analysis of efficiencies and important operating parameters," *Machines. Technologies. Materials.*, vol. 11, no. 10, pp. 481-484, 2017.

[16] Z. Zhou, M. B. Camara, and B. Dakyo, "Coordinated power control of variable-speed diesel generators and lithium-battery on a hybrid electric boat," *IEEE Transactions on Vehicular Technology*, vol. 66, no. 7, pp. 5775-5784, 2017. https://doi.org/10.1109/TVT.2016.2638878

[17] F. Aghili, "Optimal feedback linearization control of interior PM synchronous motors subject to time-varying operation conditions minimizing power loss," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 7, pp. 5414-5421, 2017. https://doi.org/10.1109/TIE.2017.2784348

[18] S. German-Galkin and D. Tarnapowicz, "Energy optimization of ship's shaft generator with permanent magnet synchronous generator," *NAŠE MORE: Znanstveni Časopis Za More i Pomorstvo*, vol. 67, no. 2, pp. 138-145, 2020. https://doi.org/10.17818/NM/2020/2.6
[19] S. German-Galkin and D. Tarnapowicz, "Optimization of the electric car's drive system with PMSM," in *Proceedings of the 12th International Science-Technical Conference Automotive Safety, Kielce, Poland*, 2020, pp. 1-6.
[20] R. Zaccone, U. Campora, and M. Martelli, "Optimisation of a diesel-electric ship propulsion and power generation system using a genetic algorithm," *Journal of Marine Science and Engineering*, vol. 9, no. 6, p. 587, 2021. https://doi.org/10.3390/jmse9060587
[21] M. Acanfora, F. Balsamo, M. Fantauzzi, D. Lauria, and D. Proto, "Load levelling through storage system for hybrid diesel electric ship propulsion in irregular wave conditions," in *Proceedings of the 2022 International Symposium on Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM), Sorrento, Italy*, 2022, pp. 640-644.
[22] M. Rezkallah *et al.*, "Coordinated control strategy for hybrid off-grid system based on variable speed diesel generator," *IEEE Transactions on Industry Applications*, vol. 58, no. 4, pp. 4411-4423, 2022. https://doi.org/10.1109/TIA.2022.3174825
[23] Z. Zwierzewicz, D. Tarnapowicz, S. German-Galkin, and M. Jaskiewicz, "Optimal control of the diesel–electric propulsion in a ship with PMSM," *Energies*, vol. 15, no. 24, p. 9390, 2022. https://doi.org/10.3390/en15249390
[24] A. Dhooge, W. Govaerts, and Y. A. Kuznetsov, "MATCONT: A MATLAB package for numerical bifurcation analysis of ODEs," *ACM Transactions on Mathematical Software*, vol. 29, no. 2, pp. 141-164, 2003. https://doi.org/10.1145/779359.779362
[25] A. Dhooge, W. Govaerts, Y. A. Kuznetsov, W. Mestrom, and A. Riet, "Cl_matcont: A continuation toolbox in Matlab," in *Proceedings of the 2003 ACM Symposium on Applied Computing*, 2003, pp. 161-166.
[26] Y. A. Kuznetsov, *Elements of applied bifurcation theory*. New York: Springer, 1998.
[27] Y. A. Kuznetsov, *Five lectures on numerical bifurcation analysis*. Utrecht, Netherlands: Utrecht University, 2009.
[28] W. J. Govaerts, *Numerical methods for bifurcations of dynamical equilibria*. Philadelphia, PA: SIAM, 2000.
[29] L. Sridhar, "Elimination of oscillation causing Hopf bifurcations in engineering problems," *Journal of AppliedMath*, vol. 2, no. 5, p. 1826, 2024. https://doi.org/10.59400/jam1826
[30] A. Flores-Tlacuahuac, P. Morales, and M. Rivera-Toledo, "Multiobjective nonlinear model predictive control of a class of chemical reactors," *Industrial & Engineering Chemistry Research*, vol. 51, no. 17, pp. 5891-5899, 2012. https://doi.org/10.1021/ie201742e
[31] W. E. Hart *et al.*, *Pyomo-optimization modeling in python*. Berlin: Springer, 2017.
[32] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25-57, 2006. https://doi.org/10.1007/s10107-004-0559-y
[33] M. Tawarmalani and N. V. Sahinidis, "A polyhedral branch-and-cut approach to global optimization," *Mathematical Programming*, vol. 103, no. 2, pp. 225-249, 2005. https://doi.org/10.1007/s10107-005-0581-8
[34] L. N. Sridhar, "Coupling bifurcation analysis and multiobjective nonlinear model predictive control," *Austin Chemical Engineering*, vol. 10, no. 3, pp. 1-7, 2024.