Research Regarding the Determination of the Power Consumed By Viscous Friction between the Rotors and the Case at a Rotating Volumetric Pump

Băran Nicolae1 --- Tănase Elena Beatrice2 --- Constantin Mihaela3* --- Mlisan (Cusma) Rasha4

1,2,3,4Faculty of Mechanical and Mechatronics Engineering, Politehnica University of Bucharest, Bucharest, Romania

Abstract

In the first part of the paper the power consumed to overcome the viscous friction between the rotors and the case in case of a rotating volumetric pump is calculated. An experimental stand used to test this new type of volumetric pump with rotors profiled is presented; the power at the working machine couple is determined and an energy balance of the installation is issued.

Keywords: Rotating volumetric pump, Profiled rotors, Consumed power, Viscous friction, Flow rate computation, Driving power computation.

Contents

1. Introduction .................................................................................................................................................. 16
2. The Constructive Solution and the Operating Principle of the Volumetric Pump with Two Special Profiled Rotors ....16
3. Computation of the Power Consumed by Viscous Friction between the Rotors at the Machine Case .........................16
4. Computation of the Power Consumed to Overcome the Hydraulic Resistance .................................................18
5. Computation of the Power Consumed to Overcome the Hydrostatic Load ......................................................19
6. Experimental Researches Regarding the Determination of the Power Consumed by Viscous Friction between the Rotors and the Case ..................................................................................................19
7. Computation of the Power Consumed to Overcome the Hydrostatic Load and the Hydraulic Resistance ................20
8. The Energy Balance Experimental of the Installation Elaboration ...........................................................................21
9. Conclusions ..................................................................................................................................................21
References .........................................................................................................................................................21

* Corresponding Author
2. The Constructive Solution and the Operating Principle of the Volumetric Pump with Two Special Profiled Rotors

The machine (fig.1) has two identical profiled rotors (2, 5) of special shape which rotate with the same speed within a case (1, 4). The synchronous rotation of the rotors is provided by two gearwheels attached to the shafts 7 and 8, which form a cylindrical gear mounted outside the machine.

Fig.1. The operating principle of the rotating volumetric machine

1- lower case; 2- lower rotor; 3- suction chamber; 4- upper case; 5- upper rotor; 6- rotating piston; 7- driven shaft; 8- discharge chamber; 9- driving shaft; 10- cavity in which the upper rotor piston enters

The aspirated fluid (fig. 1. a) is transported to the discharge and after a 90° rotation of both rotors, the situation in fig. 1. b and thereafter in Figure 1. c is reached.

2.1. The Flow Rate Computation Relations

After a 180° rotation the fluid contained in the useful volume \(V_u\) (Fig. 1. c.) is in the space between the pistons, the lower case (1) and lower rotor (2), will be sent to the discharge chamber. On a full rotation of the shaft (9) two such volumes will be transported from the suction to the discharge [2], [3]:

\[
V_u = 2 \left( \frac{\pi R_r^2}{2} + \frac{\pi R_c^2}{2} \right) l \quad [m^3/rot]
\]

(1)

The case radius \((R_c)\) is the sum of the rotor radius \((R_r)\) and the piston height \((z)\):

\[
R_c = R_r + z \quad [m]
\]

(2)

It results:

\[
V_u = \pi dz(z + 2R_r) \quad [m^3/rot]
\]

(3)

The fluid volumetric flow rate discharged by a single rotor of length \(l\) [m] and speed \(n_r\) [rot/min] is:

\[
\dot{V}_u = \pi dz(z + 2R_r) \cdot \frac{n_r}{60} \quad [m^3/s]
\]

(4)

Because the machine has two identical rotors the fluid flow rate circulated by machine will be:

\[
\dot{V}_m = 2\dot{V}_u = \pi dz(z + 2R_r) \cdot \frac{n_r}{30} \quad [m^3/s]
\]

(5)

2.2. The Machine Driving Power Computation Relations

The theoretical power of the machine is given by the relation:

\[
P = V_m \cdot \Delta p \quad [W]
\]

(6)

\[
P = \pi dz(z + 2R_r) \cdot \frac{n_r}{30} \cdot \Delta p \quad [W]
\]

(7)

From the relation (7) it is noted that the machine power varies according to the following parameters:

* Constructive parameters: \(l\) - rotor length [m]; \(R_r\) - rotor radius [m]; \(z\) - rotating piston height [m]

* Functional parameters: \(n\)-machine speed [rot/min]; \(\Delta p\)-increase pressure achieved by the pump between the suction and discharge.

3. Computation of the Power Consumed by Viscous Friction between the Rotors at the Machine Case

3.1. Computation Steps

This calculation is performed successively in two steps:

I) For the viscous friction between rotors and front walls of the case; the rotor radius is specified: \(R_r = 50 \cdot 10^{-3}\) m.

II) For the radial viscous friction between the piston top and the cylindrical case; the case radius is adopted: \(R_c = 80 \cdot 10^{-3}\) m.
Are known [4] [5] [6]:
- Water dynamic viscosity: \( \eta = 10.4 \cdot 10^{-4} \text{Ns/m}^2 \)
- Water density: \( \rho = 1000 \text{kg/m}^3 \)
- Water kinematic viscosity: \( \nu = 1.04 \cdot 10^{-6} \text{m}^2/\text{s} \)
- The hydrostatic load: \( H = 1.5 \text{m} \)
- Working machine speed: \( n_r = 200 \text{rot/min} \)

\[
\omega = \frac{2m}{60} \cdot \frac{m}{30} = \frac{\pi \cdot 200}{30} = 20.93 \text{rad/s} \tag{8}
\]

\( \bullet \) For the first stage the number \( \text{Re} \) is calculated [4]:

\[
\text{Re} = \frac{\omega R^2}{\nu} = \frac{20.93 \cdot 0.05^2}{1.046 \cdot 10^{-6}} = 50000 \tag{9}
\]

So, \( \text{Re} > 2320 \), turbulent flow results.

\( \bullet \) In the second stage:

\[
\text{Re} = \frac{\omega R^2}{\nu} = \frac{20.93 \cdot 0.08^2}{1.046 \cdot 10^{-6}} = 128000 \tag{10}
\]

\( \text{Re} > 2320 \), turbulent flow results.

### 3.2. Computation of the Power Consumed To Overcome the Frontal Friction

Between the rotor and the case vortices and "reverse flow" appear, as a result, the shear is not proportional to the rotor tangential velocity, but with its square; ie the velocity distribution in the boundary layer is not linear but quadratic, similar to a parabola.

The shear stress in turbulent regime is assimilated with a local pressure loss.

At a distance \( r \), the stress is

\[
\tau(r) = \frac{\rho \nu^2}{2} = \frac{\rho \nu (\omega r)^2}{2} \tag{11}
\]

Fig. 2. Rotating disk located in a case containing fluid

The two front surfaces (Fig. 2) of one rotor are taken into account. For the two circular rings (Fig. 3) corresponding to the radius \( r \), the elemental friction force is:

\[
dF = \tau(r) \cdot 2(2\pi dr) = \rho \nu (\omega r)^2 \cdot 4\pi dr \tag{12}
\]

and the moment compared to the shaft is:

\[
dM = r dF \tag{14}
\]

\[
dM = 2\pi \rho \nu \omega^2 r^4 dr \tag{15}
\]

Fig. 3. Computing notations

The coefficient \( \xi \) is a function of the Reynolds number for each ring, but a medium \( \xi \) is considered, function of the Reynolds number of the entire disc. Since the turbulent flow between the rotor and the case is determined by the ratio \( \rho / D \), results that: \( \xi = f(\text{Re}, \rho / D) \).

The frontal friction forces moment for one rotor will be:

\[
M_{f.r} = \int_{r_2}^{D} dM = \int_{r_2}^{D} 2\pi \rho \nu \omega^2 r^4 dr \tag{16}
\]

\[
M_{f.r} = 2\pi \rho \nu \omega^2 \left( \frac{D}{2} \right)^5 \tag{17}
\]

For the entire machine, which has two rotors, the frontal friction forces moment will be:

\[
M_{f.r.w} = 2M_{f.r} = \frac{4\pi \rho \nu \omega^2 \left( \frac{D}{2} \right)^5}{5} \tag{18}
\]

The power consumed by viscous friction for the entire machine will be Băran, et al. [9]:

\[
P_{f.w} = M_{f.r.w} \cdot \omega = \frac{4\pi \rho \nu \omega^3 \left( \frac{D}{2} \right)^5}{5} [W] \tag{19}
\]

\[
P_{f.w} = 4 \cdot \pi \cdot 1000 \cdot 20.93 \left( \frac{0.10}{2} \right)^5 = 7.196 \cdot 20.93 [W] \tag{20}
\]
3.3. Computation of the Power Consumed To Overcome the Radial Friction

The viscous friction between the piston top and the case, called radial friction can be calculated with the relation [7]:

\[ F = \tau A = \tau \pi Dl \quad [N] \] (21)

\[ F = \rho \zeta \left( \frac{\omega D}{2} \right)^3 \pi Dl = \rho \zeta \pi \omega^3 \left( \frac{D}{2} \right)^3 l \] (22)

\[ M_{fr,r} = F \frac{D}{2} = \rho \zeta \pi \omega^3 \left( \frac{D}{2} \right)^4 \] (23)

\[ M_{fr,r} = \rho \zeta \pi \omega^3 \left( \frac{D}{2} \right)^4 \] (24)

For the entire machine, which has two rotors:

\[ M_{fr,r,m} = 2M_{fr,r} = 2\rho \zeta \pi \omega^3 \left( \frac{D}{2} \right)^4 [N \cdot m] \] (25)

The power consumed to overcome the radial friction will be:

\[ P_{r,m} = M_{fr,r,m} \cdot \omega \quad [W] \] (26)

\[ P_{r,m} = 2\rho \zeta \pi \omega^3 \left( \frac{D}{2} \right)^4 [W] \] (27)

\[ P_{r,m} = 2 \cdot 1000 \cdot \zeta \cdot \pi \cdot 20.93 \cdot 0.05 \left( \frac{0.16}{2} \right)^4 = 117747 \cdot \zeta \quad [W] \] (28)

So, the power consumed to overcome the frontal and the radial friction for the entire machine will be:

\[ P_f = P_{fr,m} + P_{r,m} = 7.1968 \cdot \zeta + 117747 \cdot \zeta \quad [W] \] (29)

\[ P_f = 125.1157 \cdot \zeta \quad [W] \] (30)

3.4. The Local Pressure Losses Coefficient Computation (Z)

A sudden narrowing of the flow section is considered (Figure 4) for which the value of \( \zeta \) is determined [10], [11]:

\[ \zeta = 0.5 \left[ 1 - \frac{A_0}{A_1} \right]^{3/4} + \zeta_d \]

Fig. 4. Computing notations

Narrowed section:

\[ A_0 = l \cdot s = 50 \cdot 10^{-3} \cdot 0.01 \cdot 10^{-3} = 0.5 \cdot 10^{-6} \quad [m^2] \] (31)

\[ A_1 = z \cdot l \quad [m^2] \] (32)

\[ A_0 = 30 \cdot 10^{-3} \cdot 50 \cdot 10^{-3} = 1500 \cdot 10^{-6} \quad [m^2] \]

\[ \frac{A_0}{A_1} = \frac{0.5 \cdot 10^{-6}}{1500 \cdot 10^{-6}} = 0.00033 \] (33)

\[ \zeta = 0.5 + \zeta_d; \zeta_d = \lambda \frac{l_0}{D} \] (34)

\( l_0 \) - narrowed section length

\( l_0 = 2 \) mm; \( D = 160 \) mm

The value of \( \lambda \) [4]: \( \lambda = f (Re, D/d) \).

Re number for the flow in the channel between the rotor and the stator: \( Re = 8.8 \cdot 10^4 \)

\[ \frac{D}{D} = \frac{80}{0.03} = 2666.6 \] (35)

From Băran and Stanciu [4] it results \( \lambda = 0.02 \)

\[ \zeta_d = 0.02 \cdot \frac{2 \cdot 10^{-3}}{160 \cdot 10^{-3}} = 0.00025 \]

So: \( \zeta = 0.5 + 0.00025 = 0.50025 \)

\[ P_f = 125.1157 \cdot \zeta = 125.1157 \cdot 0.50025 = 62.589 \quad [W] \] (36)

4. Computation of the Power Consumed to Overcome the Hydraulic Resistance

4.1. Linear Pressure Loss Calculation

The following data are known:

- The fluid flow rate through the pipe \( \Phi 50 \times 3 \) mm when the machine speed is 200 rot/min:

\[ \frac{D}{l} = \frac{80}{0.03} = 2666.6 \]
\( V = 4.08 \cdot 10^{-3} \ m^3/ s \)
- The fluid velocity in the pipe:
\( w = 2.68 \ m/s \)

- Dynamic and kinematic viscosity of water [4]:
\( \eta = 10.4 \cdot 10^{-4} \ \text{Ns/m}^2; \nu = 1.04 \cdot 10^{-6} \ \text{m}^2/ \text{s} \)

- The machine speed: \( \pi r = 200 \ \text{rot/min} \rightarrow \omega = 20.93 \ \text{rad/s} \)
- Absolute roughness of the pipe walls [4]:
\( \varepsilon = 0.03 \ \text{mm} \)

The linear pressure losses [4]:

\[
\Delta p_l = \frac{\lambda}{d} \rho \frac{w^2}{2} \left[ \text{N/m}^2 \right] \quad (37)
\]

\[
\lambda = f\left( \frac{d}{\varepsilon}, \frac{\rho w}{\nu} \right) \quad (38)
\]

\[
\text{Re} = \frac{w d}{\nu} = 2.68 \cdot 0.044 = 1.13 \cdot 10^4
\]

\[
d = \frac{44}{0.03} = 1466
\]

\[
\Delta p_{lin} = 0.021 \cdot \frac{6}{0.044} \cdot 1000 \cdot \frac{2.68^2}{2} = 10283.89 \ N/ \text{m}^2
\]

The power consumed to overcome the linear hydraulic resistance will be:

\[
P_{lin} = V \cdot \Delta p_{lin} = 4.08 \cdot 10^{-3} \cdot 10283.89 = 42 \ [W] \quad (39)
\]

4.2. Local Pressure Loss Calculation

The calculation formula is Băran and Stanciu [4]:

\[
\Delta p = \sum_{i=1}^{N} \xi_i \rho \frac{w^2}{2} \left[ \text{N/m}^2 \right]
\]

\[
\sum \xi_i = \xi_{aspira} + 3 \xi_{local} + \xi_{hubmerc} + 2 \xi_{robinet}
\]  

From Idelcik [11] it results:

\[
\sum \xi_i = 1.6 + 3 \cdot 0.3 + 2.1 + 2 \cdot 1 = 6.6
\]

\[
\Delta p_{loc} = 6.6 \cdot 1000 \cdot \frac{2.68^2}{2} = 23701.92 \left[ \text{N/m}^2 \right]
\]

The power consumed to overcome the local resistance will be:

\[
P_{loc} = V \cdot \Delta p_{loc} = 4.08 \cdot 10^{-3} \cdot 23701.92 = 96.7 \ [W] \quad (41)
\]

The total power consumed to overcome the hydraulic resistance linear+local will be:

\[
P_{loc} = P_{lin} + P_{loc} = 42 + 96.7 = 138.7 \ [W] \quad (42)
\]

5. Computation of the Power Consumed to Overcome the Hydrostatic Load

The calculation formula is Holman [12], Exarhu [13]:

\[
P_H = V \cdot \Delta p_H \ [W]
\]

\[
\Delta p_H = \rho g h = 10^3 \cdot 9.81 \cdot 1.5 = 14715 \ \text{N/m}^2
\]

\[
P_H = 4.08 \cdot 10^{-3} \cdot 14715 = 60 \ [W] \quad (44)
\]

6. Experimental Researches Regarding the Determination of the Power Consumed by Viscous Friction between the Rotors and the Case

6.1. The Experimental Installation Scheme

Figure 5 shows the experimental installation scheme constructed in closed circuit [14].

The working fluid (air, water, oil) is absorbed from the tank (2) by the rotating pump with profiled rotors (5). This machine has the advantage that it can circulate both gas (air) as well as liquids (water, oil).

- The pump is driven by an anti-explosive electrical motor (6) whose speed is controlled by a frequency converter (7).
- At the pump discharge, the fluid passes through an electromagnetic flow meter (10) and a flow control valve (11). Thereafter, the fluid is pumped to h = 2 m and finally arrives in the tank (2).
- In the tank, the fluid layer has a height of 0.5 m, height that provides the fact that the rotating pump with profiled rotors will always be “flooded” (fig. 5).
This type pump of circulates any fluid, but the electromagnetic flow meter puts the condition: the electrical conductivity of the fluid subjected to the measurement should be greater than 200 μS/m; as a result for the circulated flow rates and subject to the measurements it will refer only to water.

The pipeline route through which the fluid flows is made of transparent Plexiglas Ø 50x3 mm, which permit a good view of the flow. On the circulation pipeline route of the fluid, pressure gauges, thermometers, electromagnetic flowmeter are located; the pump speed can be changed with an electrical current frequency converter [15], [16].

6.2. Experimental Researches Objectives

As a result of the voltage (U) and three-phase electric current intensity (I) measurement, the electrical power absorbed by the electric motor is determined:

\[ P_{me} = \sqrt{3} U \cdot I \cdot \cos \varphi \text{[W]} \]  (45)

From the electrical motors factory catalog, for the chosen motor, are given [17]:
- Electrical motor efficiency: \( \eta_{me} = 0.747 \);
- Power factor: \( \cos \varphi = 0.71 \)

The power consumed by the electric motor is calculated \( P_{me} \) and by calculation, taking into account the electric motor efficiency \( \eta_{me} \), the power at the machine couple \( P_{cm} \) is determined.

\[ P_{cm} = P_{me} \cdot \eta_{me} \text{[W]} \]  (46)

Subsequently, from \( P_{cm} \) the power consumed to overcome the hydrostatic pump load and the hydraulic linear and local resistance \( P_{\Delta p} \) on the installation circuit is subtracted; the remaining value must be equal to the power consumed by viscous friction \( P_{t,f} \).

With the equations (45) and (46), based on the experimental data \( U, I \), the data in Table 1 resulted, the working fluid was water.

Table 1. Values of \( P_{me} \) and \( P_{cm} \) for water

7. Computation of the Power Consumed to Overcome the Hydrostatic Load and the Hydraulic Resistance

7.1. Computation of the Power Consumed to Overcome the Hydrostatic Load [12], [13]

For exact calculations \( n_r = 200 \text{ rot/min} \) is chosen.

\[ P_H = \dot{V} \cdot \Delta p_H \text{[W]} \]  (47)

where:
- \( \dot{V} \): volumetric flow rate \([\text{m}^3/\text{s}]\)
- \( \Delta p_H \): pressure increase \([\text{N/m}^2]\)

\[ \Delta p_H = \rho g H = 10^3 \cdot 9.81 \cdot 1.5 = 14715 \text{ N/m}^2 \]

\[ P_H = 4.08 \cdot 10^{-3} \cdot 14715 = 60 \text{[W]} \]  (48)

7.2. Computation of the Power Consumed to Overcome the Hydraulic Resistance from the Circuit

a) Linear pressure loss calculation

This calculation is performed when \( n_r = 200 \text{ rot/min} \).

The following data are known:
- The fluid flow rate through the pipe Ø 50 x 3 mm:
  \[ \dot{V} = 4.08 \cdot 10^{-3} \text{ m}^3/\text{s} \]
- The fluid velocity (water) in the pipe:
  \[ w = 2.68 \text{ m/s} \]
- Dynamic and kinematic viscosity of water [4]:
  \[ \eta = 10.4 \cdot 10^{-4} \frac{\text{N} \cdot \text{s}}{\text{m}^2} ; v = 1.04 \cdot 10^{-6} \text{ m}^2/\text{s} \]
- Absolute roughness of the transparent plexiglas pipe walls [4]:
  \[ \varepsilon = 0.03 \text{ mm} \]

The linear pressure losses, are given by relation [4, 5]:

\[ \Delta p_L = \frac{f \cdot \dot{V} \cdot w^2}{d \cdot \rho \cdot 2} \text{[N/m}^2]\]  (49)

where:
\[ f = f \left( \frac{\text{Re}}{\varepsilon} \right) \]
\[
\text{Re} = \frac{wd}{\nu} = \frac{2.68 \cdot 0.044}{1.04 \cdot 10^{-6}} = 1.13 \cdot 10^{5}
\]

From Băran and Stanciu [4] it results: \( \lambda = 0.021 \)

\[
d = \frac{44}{0.03} = 1466
\]

\[
\Delta p_{\text{lin}} = 0.021 \cdot 1000 \cdot \frac{2.68^2}{2} = 10283.89 \text{N/m}^2
\]

The power consumed to overcome the linear hydraulic resistance will be:

\[
P_{\text{lin}} = V \cdot \Delta p_{\text{lin}} = 4.08 \cdot 10^{-3} \cdot 10283.89 = 42 \text{[W]}
\]

b) Local pressure loss calculation

In Băran and Stanciu [4] the relation is specified:

\[
\Delta p_{\text{loc}} = \sum \xi_i p_i \frac{w_i^2}{2} \left[ \frac{N}{m^2} \right] \quad (50)
\]

\[
\sum \xi_i = \xi_{\text{pump suction}} + 3 \xi_{\text{elbow 90°}} + \xi_{\text{valve}} + 2 \xi_{\text{valve}} \quad (51)
\]

From Kiselev [10], Idelcik [11] it results:

\[
\sum \xi_i = 1.6 + 3 \cdot 0.3 + 2.1 + 2 \cdot 1 = 6.6
\]

\[
\Delta p_{\text{loc}} = 6.6 \cdot 1000 \cdot \frac{2.68^2}{2} = 23701.92 \left[ \frac{N}{m^2} \right]
\]

The power consumed to overcome the local resistance will be:

\[
P_f = V \cdot \Delta p = 4.08 \cdot 10^{-3} \cdot 23701.92 = 96.7 \text{[W]}
\]

The total power consumed to overcome the hydraulic resistance (linear+local) will be:

\[
P_{\text{hp}} = P_{\text{lin}} + P_{\text{loc}} = 42 + 96.7 = 138.7 \text{[W]}
\]

8. The Energy Balance Experimental of the Installation Elaboration

Following the experimental researches the volumetric pump driven power resulted (the power at the machine couple): \( P_{\text{on}} = 264.93 \) [W]

From this value are deducted:

- The power consumed to overcome the hydrostatic load: \( P_{\text{h}} = 60 \) W
- The power consumed to overcome the hydraulic resistance from the circuit: \( P_{\text{hp}} = 138.7 \) W

\[
P_{\text{on}} = (P_{\text{h}} + P_{\text{hp}}) = 264.93 - (60 + 138.7) = 66.23 \text{[W]}
\]

The theoretical calculated value regarding the power consumed by viscous friction between the rotors and the case was \( P_f = 62.589 \) W; the difference between theory and experiment is:

\[
\Delta P = 66.23 - 62.58 = 3.65 \text{[W]}
\]

This difference contains the power consumed in the machine bearings and other losses.

The effective efficiency of the rotating volumetric pump will be:

\[
\eta_e = \frac{P_{\text{hp}} + P_{\text{hp}}}{P_{\text{on}}} = \frac{60 + 138.7}{264.93} = 0.75
\]

This value is very good compared to data from specialty literature [18], [19].

9. Conclusions

- The computing results established for the determination of the power consumed by viscous friction between the rotors and the case, for a turbulent flow regime, indicates a good coincidence with the experimental data.
- From the received power at the machine couple (264.93 W) one part (60W), about 22% is consumed to overcome the hydrostatic load, a part (138.7 W), about 52% is consumed to overcome the hydraulic resistances in the pump circuit and the rest to overcome the viscous friction between the rotors and the case (62.58 W), ie 24%, and 2% are other losses.
- The effective efficiency of the volumetric pump, calculated for a speed of 200 rot / min, indicates a value of 75%, which is superior to other pumps types.

References


Views and opinions expressed in this article are the views and opinions of the authors, Asian Engineering Review shall not be responsible or answerable for any loss, damage or liability etc. caused in relation to/arising out of the use of the content.